Fun With Sets

The goal of this assignment is to get some practice with identifying and using sets. Recall that $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ is the set of natural numbers and $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ is the set of integers. If $S$ is a set, then $|S|$ is the cardinality of that set. We also denote by $[n]$ the set $\{1, 2, \ldots, n\}$, but in the following exercises, please write out the longer form.

The standard presentation of a set is to list its elements inside braces, such as $\{1, 2, 3\}$. When the list of elements is large (or infinite), the use of ellipses ($\ldots$) is okay, such as for $\mathbb{N}$ or $\mathbb{Z}$ above. Alternatively, inside the braces, we adopt the convention that a vertical line “|” means “such that” and a comma “,” means “and”.

In the following exercises, if a task is not specified, the implicit task is to write out the elements of the sets in the following exercises. Here are some examples of your task:

- $\{n \in \mathbb{N} \mid n > 2, n < 6\} =$
  $\{3, 4, 5\}$
- $\{n \in \mathbb{N} \mid n > 17\} =$
  $\{18, 19, 20, 21, \ldots\}$
- $\{2^n \mid n \in \mathbb{N}, n \geq 3\} \cap \{n \in \mathbb{N} \mid n < 18\} =$
  $\{8, 16\}$
- $\{[2], [5]\} =$
  $\{\{1, 2\}, \{1, 2, 3, 4, 5\}\}$

Exercises:
1. $(\{n \in \mathbb{Z} \mid n \text{ is divisible by } 2\} \cap \mathbb{N}) \cup \{-5\} =$
2. $\{(1, n) \mid n \in \mathbb{N}, 5 < n < 10\} =$
3. $\{(2n, -n) \mid n \in \mathbb{Z}, -2 < n < 4\} =$
4. $(a, b) \mid a \in \{F, G, H\}, b \in \{5, 8, 9\} =$
5. $\{F, G, H\} \times \{5, 8, 9\} =$
6. $(a, b) \mid a, b \in \{7, 8, 9\}, a < b =$
7. The following set has elements which are themselves sets.
   $\{[n] \mid n \in \mathbb{N}, 1 \leq n \leq 3\} =$
8. \( \{ (1, 2, \ldots, n) \mid n \in \mathbb{N}, n \leq 5 \} = \)
9. \( \{ (x, y) \in \mathbb{N} \times \mathbb{N} \mid y = 2x, x = 2y \} = \)
10. \( \{ (x, y) \in \mathbb{N} \times \mathbb{N} \mid y = 2x, 1 \leq x \leq 4, y < 8 \} = \)
11. \( (\{1, 2\} \times \{1, 2\}) \times \{1, 2\} = \)
12. \( (\{1, 2\} \cup \{1, 2\}) \cup \{1, 2\} = \)
13. \( \{1\} \cup \{1, 2\} \cup \{1, 2, 3\} \cup \ldots = \)
14. \( \{ \{a\} \cup \{b\} \mid a \in \mathbb{N}, b \in \mathbb{N}, 1 \leq a \leq 4, 3 \leq b \leq 5 \} = \)
15. \( \{ X \mid X \subset [4], |X| \neq 3 \} = \)
16. \( [ |3| ] = \)
17. \( \{ \{12\} \} \cup \{12\} = \)
18. \( \{ [k] \mid k = |S \times \{1, 2\}|, S \subset [3] \} = \)
19. Set \( A = \{1, 2\}, B = \{3, 4\}, \) and \( f = \{ (a, b) \mid a \in A, b \in B \}. \)
Write out the elements of \( f. \) Is \( f \) a function \( A \rightarrow B? \)

20. Let \( C = \{ (1, 2), (2, 2), (3, 2) \}. \)
Can \( C \) be a function? (For starters, what would \( A \) and \( B \) be?)

Write out the elements of the following set \( D. \)
\( D = \{ (b, a) \mid (a, b) \in C \} = \)

Can \( D \) be a function?