MATH 277X INTRODUCTION

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This course will explore the applications of the theory of *foliations and laminations* to 3–manifold topology, and in particular, the relationship between the theory of *essential laminations* and *geometric structures* on 3–manifolds.

An essential lamination has at least three secret lives:

- A topological life. An essential lamination is a generalization of an incompressible surface. There are analogies with the structure theory of Haken manifolds, hierarchies, etc. Like incompressible surfaces, essential laminations behave reasonably well with respect to triangulations, and broaden the domain of the theory of normal surfaces in many nontrivial ways.
- A dynamical life. Taut foliations are characterized by the existence of transverse volume–preserving flows. More generally, the transverse space of a lamination in a 3–manifold has a natural (local) order structure. The simplest foliations — surface bundles over circles — give a lot of information about automorphisms of compact surfaces. Finite depth foliations are related to end–periodic automorphisms of infinite surfaces. Transverse pseudo–Anosov flows are generalizations of extremal quasiconformal maps between compact surfaces.
- An analytic life. Surface laminations are generalizations of Riemann surfaces, and one can define a Teichmüller space, quadratic differentials, mapping class elements etc. for a lamination. In this context there are analogues of the uniformization theorem, projective embeddings, and other classical results.

Revealing and relating these hidden facets will be a major goal of the course. Topics to be covered may include the following:

- Basic definitions and properties of codimension one foliations
- Haefliger structures and existence results
- Numerical invariants of foliations
- Reebless foliations and Novikov's theorem
- Sullivan's theorem and the min. cut — max. flow principle
- Foliations of circle bundles and Seifert fibered spaces
- Finite depth foliations
- Algebraic and analytic geometry of surface laminations
- Genuine laminations and the weak geometrization conjecture
- Groups of homeomorphisms of the circle
- Slitherings
- Pseudo–Anosov packages
- Non–existence results
- Coarse foliations and generalized foliations

The course home page is at the URL

`http://www.math.harvard.edu/~dannyc/courses/fol/fol.html`

References, links and material for download are available there.

The textbook for the course is *Foliations I* by Candel and Conlon. It's an AMS Graduate studies in Mathematics volume, number 23, and is available at the Co–op.

Here are some motivating problems to think about. If you think these problems are interesting, you might enjoy this class.

Problem 1. *Find a smooth, properly embedded plane π in $\mathbb{R}^3$ such that there is no homeomorphism $\mathbb{R}^3 \to \mathbb{R}^3$ taking π to a horizontal plane.*

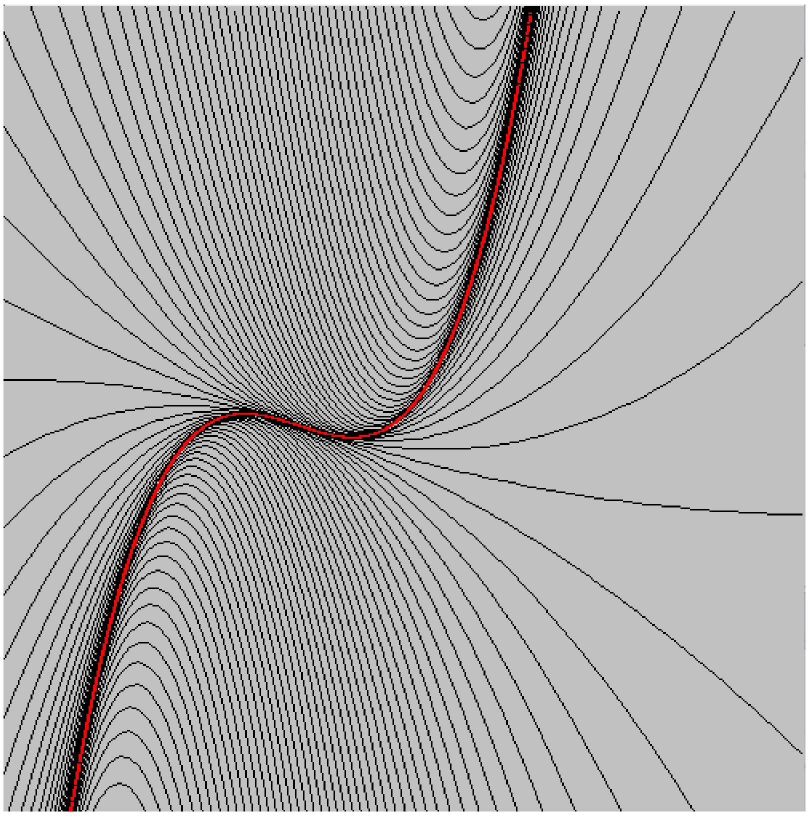
Problem 2. *Let M be a 3–manifold with boundary a countably infinite union of planes, divided into two infinite sets X_i^+ and X_i^-, whose interior is homeomorphic to B^3. Find a* complete *Riemannian metric on M of bounded curvature such that every point in an X_i^+ is a uniformly bounded distance from some point in an X_j^-, and vice versa.*

Problem 3 (Meeks)**.** *Let π be a properly embedded minimal surface contained in a half–space in Euclidean 3–space. Show π is a flat plane. This is in sharp contrast to a minimal surface in hyperbolic 3–space.*

Problem 4. *Let G be a group of orientation–preserving homeomorphisms of $\mathbb{R}$ such that each element has at most one fixed point, and suppose further that G does not have a global fixed point. Show that G is conjugate to a subgroup of the group of* affine *transformations of $\mathbb{R}$.*

Problem 5. *Let $\mathscr{F}$ be a foliation of M a hyperbolic 3–manifold by closed surfaces, and X a transverse vector field. Show that the flowlines of $\widetilde{X}$ (on the universal cover $\widetilde{M} = \mathbb{H}^3$) are uniformly quasigeodesic.*

Problem 6. *Let G be the group of homeomorphisms of the circle. What properties does G have as an abstract discrete group? What is the homology of this group?*



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