Errata, clarifications, and updates for
***Algorithmic Randomness and Complexity*,**
by Rodney G. Downey and Denis R. Hirschfeldt

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For the latest version of this list, see `http://math.uchicago.edu/~drh/arc.html`.

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Major corrections:

Section 13.9. Greenberg and Miller have informed us that their analysis of the construction by Kumabe and Lewis in [225] is not in fact sufficient to establish Theorem 13.9.1. Thus Theorem 13.9.17 cannot at present be established with the argument given in Section 13.9, and the existence of a minimal degree of effective Hausdorff dimension 1 is open as of the writing of this note [January 2011], as is the result claimed as Theorem 13.9.1. However, Greenberg and Miller have a new version of [170], available at `http://www.math.wisc.edu/~jmiller/Papers/slowDNR.pdf`, in which they show that for every computable order $h : \mathbb{N} \to \mathbb{N} \setminus \{0, 1\}$, there is an $f \in \mathrm{DNC}_h$ that does not compute a 1-random set. Replacing Theorem 13.9.1 by this result, the argument in Section 13.9.1 yields the most important consequence of Theorem 13.9.17: There is a set of effective Hausdorff dimension 1 that does not compute a 1-random set.

Typos and other corrections:

Section 1.2. The following corrections should be made to this section.

Page 6, lines 9 and 17: $B_\varepsilon - \bigcup_i [\![\sigma_i]\!]$ should be $B_\varepsilon \setminus \bigcup_i [\![\sigma_i]\!]$.

Page 6, line 12: The last term in the chain of inequalities should be replaced by $\leqslant (1-\varepsilon)\mu(U) < \mu^*(B_\varepsilon)$.

Page 6, line -15: $2^{-|\sigma_n|+1}$ should be $2^{-|\sigma_{n+1}|+1}$.

Section 2.12. Theorem 2.12.3: The nontriviality condition in the statement of the theorem is not needed, and is not used in the proof. In the last paragraph of the proof, t should also be chosen large enough so that if $m < n$ and $\gamma(m,t) \in A$, then $\gamma(m,t) \in A[t]$. (This condition can be checked V^A-computably.)

Section 2.16. Page 65, line 1: There is an extra 'r' in that line.

Section 2.19.1. Page 73, footnote 12, line 3: The machine being described should return 1 if $k \in W_e$ and 0 if $k \in W_j$.

Section 2.19.2. Page 76, lines 6–8: The two full sentences in these lines are incorrect. They should read as follows: "A $\Pi_1^{0,\emptyset^{(n)}}$ class is also a Π^0_{n+1} class, since every $\emptyset^{(n)}$-computable relation is Π^0_{n+1}. Similarly, every $\Sigma_1^{0,\emptyset^{(n)}}$ class is also a Σ^0_{n+1} class."

Section 2.21. Page 85, line −15: "We have $\psi_i \in T$ iff ψ_i is true, and at most one ψ_i is true." should be "If ψ_i is true then $\psi_i \in T$, and $T \vdash \neg(\psi_0 \wedge \psi_1)$."

Section 3.4. Loveland [250] attributes Theorem 3.4.1 to A. R. Meyer.

Section 3.5. Page 124, line 2: The τ should be a σ.

Section 3.6. The following typos occur in this section.

> Page 126, line −18: $2^{-|\tau|}$ should be 2^{-d}.
>
> Page 127: In the displayed equation in the proof of Proposition 3.6.4, $k \leqslant -\log f(n)$ should be $k \geqslant -\log f(n)$.

Section 5.1. Page 199, line 9: "the" should be "then".

Section 6.1. Proposition 6.1.2: The proof as given shows only that $\Omega \equiv_{\mathrm{T}} \emptyset'$. To show that $\Omega \equiv_{\mathrm{wtt}} \emptyset'$, first note that every left-c.e. real is ω-c.e., and hence wtt-computable in $\emptyset'$. For the other direction, note that the reduction given in the proof of Proposition 6.1.2 is in fact a wtt-reduction, since to find a stage s as in the proof requires only knowing the first $|\rho| + e + 2$ many bits of Ω.

Section 6.2. The following typos occur in this section.

> Page 230, line −15: $\sum_{i<n} a(i)$ should be $\frac{\sum_{i<n} a(i)}{n}$.
>
> Definition 6.2.4, line 2: $\bigcup_n U_n$ should be $\bigcap_n U_n$.

Section 6.5. The following typos occur in this section.

> Page 247, line 10: The first element of $C(6)$ is h, not f.
>
> Page 247, lines 14 and 18: $C \in E$ should be $C \subseteq E$.
>
> Page 247, line 16 (twice): $C(n_{i_j})$ should be $\alpha_{n_{i_j}}$.
>
> Page 247, line 19: The absolute value should end before the inequality.
>
> page 248, line −15: $I(n) = \ell$ should be $I(n) < \ell$.
>
> page 249, line −11: The second $2^{k+m(j)+2}$ should be $2^{k+m(j)+1}$.
>
> page 249, line −4: $-\frac{1}{2}$ should be $-\frac{n}{2}$.

Section 6.6. Page 250, line −5: The α should be an A.

Section 6.7. Page 253, line −4: $K(x \upharpoonright t) \leqslant t - k$ should be $K(A \upharpoonright t) \leqslant t - k$.

Section 7.2. Page 287, lines −16 and −14: "weakly 1-random" should be "weakly n-random".

Section 8.2. Page 324, line -3: Here we need to assume our approximations are sufficiently sped-up so that $\Psi^B(n+k)[s]\downarrow = A_s(n+k)$ for all s and $n \leqslant s$.

Section 8.12. Page 358: In the proof of Theorem 8.12.1, the variable n is used with two different meanings. The last sentence in that proof should read as follows: "More precisely, $m \in A$ iff $2^n\mu(\{Y : \sigma \prec Y \wedge m \in W_e^Y\}) > \frac{1}{2}$, and the set of m for which this inequality holds is clearly c.e."

Section 11.1.1. Page 501, line -1: It is worth noting that we adopt the usual convention that $s > n$.

Section 11.2. Page 508, line 2 of the proof of Theorem 11.2.5: "simplicity requirements" should be "noncomputability requirements".

Section 11.4. The following are corrections to the proofs of Theorems 11.4.1 and 11.4.9. They are relatively minor, but since they involve multiple slight changes, we have also provided corrected versions of these proofs in a separate file, available at `http://math.uchicago.edu/~drh/arc2.pdf`.

> On page 520, line -11, the expression $Q_{i-1,e,u}(2^{-(e+1)}g, \min(2^{-(e+1)}g, 2^{-(2i+n)}))$ should be $Q_{i-1,e,u}(2^{-(e+1)}q, \min(2^{-(e+1)}q, 2^{-(2i+n)}))$. Consequently, on page 523, line 12, $2^{-(e+1)}g$ should be $2^{-(e+1)}q$, and hence on line 14 of the same page, 2^e should be $\frac{2^e g}{q}$. However, see the following item for a further correction to these lines.
>
> The parenthetical comment beginning at the bottom of page 520 is incorrect, because the weight of C might be very close to g, due to the enumeration into C of sets built by $Q_{i-1,e',u'}$ procedures with e' much larger than e. It might then be the case that for every subset $\widehat{D}$ of D, the weight of $C \cup \widehat{D}$ is greater than g. There are several ways to fix this issue. Perhaps the simplest is to allow procedures to return up to twice their goals by making the following adjustments:
>
> > On page 520, line -11, call $Q_{i-1,e,u}(2^{-(e+2)}q, \min(2^{-(e+2)}q, 2^{-(2i+n)}))$.
> >
> > Beginning on page 520, line -4, item 2.(a) should now read as follows: Put D into C. If the weight of C (defined as in the proof of Theorem 11.3.1) is less than g then proceed to step 2(b). Otherwise, cancel all runs of subprocedures, and end this run of P_i, returning C. (Note that in this case the weight of C is less than $2g$.)
> >
> > On page 521, line 9, call $P_j(\frac{q}{2}, \min(\frac{q}{2}, 2^{-(2j+3+n)}))$.
> >
> > On page 521, lines 12–13, the parenthetical remark should now read as follows: It is easy to see that in this case the weight of D is less than $2g$.
> >
> > On page 521, lines 17–18, the parenthetical remark should now read as follows: In this case the weight of D is equal to g.
> >
> > On page 522, line 13, replace $P_j(q, q')$ by $P_j(\frac{q}{2}, q')$.
> >
> > On page 522, beginning on line 16, item (ii) should now read as follows: Let n be one of the numbers corresponding to a run of $P_i(g, q)$ that is in D_{i-1} by a stage t but never enters C_i. Then n is put in D_{i-1} by a run of a subprocedure

$Q_{i-1,e,u}(2^{-(e+1)}q, q')$ called by this run of $P_i(g, q)$. It cannot be the case that $A_s \upharpoonright u \neq A_{s-1} \upharpoonright u$ for some $s > t$, as otherwise n would enter C_i at stage s. Thus the run of $Q_{i-1,e,u}$ that put n into D_{i-1} is never released, and e is never again available, so this run of $P_i(g, q)$ never calls a subprocedure $Q_{i-1,e,u'}$ after stage t.

Thus, for each e there is at most one run of a subprocedure of the form $Q_{i-1,e,u}(2^{-(e+2)}q, q')$ called by our run of $P_i(g, q)$ that leaves numbers in $D_{i-1} \setminus C_i$. Thus the sum of the weights of these numbers over all such e is at most $\sum_e 2^{-(e+1)}q = q$.

On page 523, line 12, $2^{-(e+1)}g$ should be "at most $2^{-(e+1)}q$", and hence on line 14 of the same page, 2^e should be $\frac{2^e g}{q}$.

The proof of Theorem 11.4.9 beginning on page 524 should be similarly corrected:

On page 524, line 14, the expression $Q_{i-1,\sigma,\tau,u}(2^{-|\sigma|}g, \min(2^{-|\sigma|}g, 2^{-(2i+n)}))$ should be $Q_{i-1,\sigma,\tau,u}(2^{-|\sigma|+1}q, \min(2^{-|\sigma|+1}q, 2^{-(2i+n)}))$.

On page 524, beginning on line 20, item 2.(a) should now read as follows: Put D into C. If the weight of C is less than g then proceed to step 2(b). Otherwise, cancel all runs of subprocedures, and end this run of P_i, returning C.

On page 524, line -8, $P_j(q, 2^{(-2j+3+n)})$ should be $P_j(\frac{q}{2}, \min(\frac{q}{2}, 2^{(-2j+3+n)}))$.

On page 525, the paragraph beginning on line 1 should now read as follows: The construction runs as before. The verification that there is a golden run is also essentially as before, except for the last paragraph of the proof of Lemma 11.4.3. In this case, the same argument as in that proof shows that for each σ, there is at most one run of a subprocedure of the form $Q_{i-1,\sigma,\tau,u}(2^{-|\sigma|+1}q, q')$ called by our run of $P_i(g, q)$ that leaves numbers in $D_{i-1} \setminus C_i$. Furthermore, none of these subprocedures are ever released, so all of the corresponding σ's are in $\operatorname{dom} \mathcal{U}^A$, and thus the sum of the weights of these numbers over all such σ is bounded by $\Omega^A q < q$.

On page 525, lines 17 and 21, the expression $Q_{i-1,\sigma,\tau,u}(2^{-|\sigma|}q, q')$ should be $Q_{i-1,\sigma,\tau,u}(2^{-|\sigma|+1}q, q')$.

On page 525, line 22, $2^{-|\sigma|}q$ should be "at most $2^{-|\sigma|}q$".

Section 14.1. Page 670, line -13: The word "well" before "sets" should be deleted.

Section 16.3.1 Page 739, line 4: "$D_{f(n)} \in B$" should be "$D_{f(n)} \subseteq B$".

Index. Some of the references to pages in the frontmatter are off by a few pages, and some words with accented characters are alphabetized incorrectly.