PROGRAM NOTES AND ABSTRACTS FOR WEEKS 5 AND 6

Abstracts are listed in order of the talks, starting in WEEK 5

Apprentice Program: Daniel Minahan

TITLE: (Monday) Classification of surfaces.

ABSTRACT: A surface is a 2-dimensional manifold. Up to the appropriate notion of equivalence, all surfaces can be described by a finite amount of information. We will explain this classification.

Apprentice Program: Andreas Stavrou

TITLE: (Tuesday) The hairy ball theorem

ABSTRACT: We will speak about winding numbers, vector fields, and the Euler characteristic and prove the hairy ball theorem. We will then aim towards the first ideas of Morse theory.

Apprentice Program: Aditya Pahuja

TITLE: Integer Partitions

ABSTRACT: For a positive integer n, a partition of n is a way to write n as a sum of a nondecreasing sequence of positive integers. These objects appear in many places in combinatorics, algebra, and number theory, and it is quite interesting to count them! I will show a combinatorial proof of Euler's pentagonal number theorem and a power series identity called the Jacobi triple product identity, and then prove a famous theorem of Ramanujan which says that the number of partitions of $5n + 4$ is always a multiple of five.

Mark Behrens

TITLE: Exotic spheres

ABSTRACT: The Poincare Conjecture in dimension n asks if any closed n-manifold M which has the same fundamental group and homology groups of the n-sphere S^n is homeomorphic to S^n. This is easy to see in low dimensions $n < 3$, and was famously proven to in fact be true in all dimensions (Smale $n > 4$), (Freedman $n = 4$), (Perelman $n = 3$). But what if we consider smooth manifolds, and ask if M is diffeomorphic to S^n (not just homeomorphic)? This is the Smooth Poincare Conjecture. Milnor showed that the smooth Poincare Conjecture is false in dimension $n = 7$. I will describe Milnor's counterexample, and more generally discuss the theory of Kervaire-Milnor which allows for the classification of these smooth manifolds up to diffeomorphism in dimensions $n > 4$. This is a story that continues to evolve in present day.

Victor Ginzburg

TiTLE: Quivers in algebra and geometry
ABSTRACT: We will discuss a famous theorem of Gabriel on quiver representations that distinguishes Dynkin graphs from other graphs. Time permitting, we will also present another result involving point counting over finite fields that remained an open conjecture for 30 years.

Tomer Schlank

TITLE: Homotopy and Rational Points

ABSTRACT: A Diophantine equation is a polynomial equation for which we seek solutions in a specified number system, such as the rational numbers. From the perspective of algebraic geometry, finding such solutions can be viewed as finding sections for maps of "Schemes"

Schemes may be regarded as generalized spaces, suggesting that methods from algebraic topology could help us study Diophantine equations. In topology, Bousfield and Kan developed a theory using algebraic topology to detect obstructions to the existence of sections of a map of spaces. In this talk, I will explain how this theory can be transferred to algebraic geometry,

Ajay Srinivasan

TITLE: Mathematical Genealogies

ABSTRACT: Mathematicians work in a world that is a consequence of its contingent social history. Our mathematical beliefs are shaped by these contingencies as to where we find ourselves in such a history. What can we learn from the contingent origins of our mathematical beliefs? This lecture series explores critical genealogies in mathematics, focusing on algebraic geometry. As a case study, we will ask: what is it about projective modules that makes them important? In pursuing this question, we will revisit the social histories of algebraic fiber bundles, category theory, homological algebra, and sheaf theory, while emphasizing the role of personal ties from the Bourbaki movement. Time permitting, we may also discuss broader topics such as the role of the Weil conjectures in legitimizing scheme-theory, denigration of vision in Bourbakian mathematics, and the Hilbert program. The series will conclude with a discussion on the pragmatic relevance of critical genealogies in mathematics. What can such studies tell us about the ways our representations and institutions function? Parts of this series are inspired by work of Aravind Asok, and conversations with him in recent years.

The goal of the series is to encourage reflection on the ways we do mathematics—and I hope it will be valuable to all irrespective of mathematical background.

Peter May

TITLE: Topics in and around algebraic topology

ABSTRACT: I will mainly talk about characteristic classes and the calculation of the cohomology of classifying spaces.

I will try to be intelligible and to keep things reasonably independent of the previous weeks' talks.

SIXTH WEEK

Aadrita Laha

TITLE: What the Fractal!!

ABSTRACT: The name Felix Klein is associated with Kleinian groups (not to be confused with the Klein four-group), which oddly enough is due to Poincaré. Poincaré used this nomenclature to settle a dispute when he named Fuchsian groups after Lazarus Fuchs, which turned out to be due to Klein. Klein, like most mathematicians, was fascinated by symmetry leading him to study what we now know as hyperbolic geometry and Möbius maps. These were later consolidated into the subject of Kleinian groups. These groups give rise to some beautiful symmetric sets, which are fractals. I will present these magnificent and elusive sets and try to illustrate some of the history and mathematics with lots of pictures and puns.

Altan Erdnigor

TiTLE: 3264

ABSTRACT: How many lines pass through two points? One. How many circles pass through three general points? Also one. How many conics pass through five general points? Again, one. These are easy. Let us ask more: How many lines meet four general lines in 3-space? Two. How many circles are tangent to three general circles in the plane? Eight.* These are less obvious, but still doable for a high school student. However, if we continue asking questions of this kind, we quickly get stuck. How many lines lie on a smooth cubic surface? Twenty-seven. How many conics are tangent to five general conics in the plane? 3264. How many twisted cubic curves are tangent to twelve general quadric surfaces in 3-space? 5819539783680. We will explain the classical a la Schubert solution to the penultimate problem and discuss a more modern moduli-space approach to enumerative geometry. *Depending on the configuration, some of the solutions may not be real. Over the complex numbers, however, the answer is always eight. Keywords: projective geometry, algebraic varieties, blow-ups, intersection theory, cohomology, moduli spaces.

Eugenia Cheng

TITLE: Duality in category theory

ABSTRACT: The dual of a category has the same objects but all the morphisms are considered to point the opposite way. This simple switch in point of view does not add or change any information and yet is a remarkably productive construction in category theory. In this talk I will show how this enables us to unify various mathematical concepts that are categorical duals of each other, including lowest common multiples and highest common factors, suprema and infima, products and disjoint unions of sets, intersections and unions, injective and surjective functions, and more. The broader message is to show how categorical thinking is a discipline that helps us to see disparate mathematical structures in a unified way. I will not assume any category theory beyond the definition of a category.

Ajay Srinivasan

TITLE: Mathematical Genealogies
ABSTRACT: Continuing

Peter May

TiTLE: Topics in and around algebraic topology
ABSTRACT: Continuing